Pricing and Hedging Tolling Agreements

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Overview

- Features of tolling contracts
- Nature of value
  - Forward and spot
  - Approximations and representation of value
- Identification of value drivers
  - Different market structures
  - Hedging programs
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Types of Tolling Contracts

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- More generally: rental or leasing agreement giving the buyer the right to a power plant’s output
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  - *E.g.*, ability to reverse previous commitments if price spreads move for/against you
- Many operational decisions take place at the level of spot prices (*e.g.* hourly dispatch)
- However, hedging typically takes place at the level of forward prices (*e.g.* monthly)
Forward vs. Spot Value

- Value can be captured on a forward basis by appropriate trading strategies
  - Delta hedging
  - Vega hedging
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- There may be opportunities for similar strategies at the spot level
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- Depends on underlying market structure
  - The two valuations are in fact related (Mahoney and Wolyniec 2012d)
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- Much more efficient way of isolating the relevant value drivers
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- $u$: state of the unit (e.g. on or off for simplicity)
General valuation problem is an optimal stochastic control problem (for an appropriate pricing measure \( Q \)):

\[
V_t(G_t, P_t, u_{t-1}) = \\
\sup_{u_t, \ldots, u_T} E_t^Q \sum_{s=t}^{T} u_s ((\alpha Z_s + (1 - \alpha) Z_s^+) - (1 - u_{s-1}) X_s)
\]
Amenable to backward induction:

\[ V_t(G_t, P_t, u_{t-1}) = \]

\[ \max_{u_t(G_t, P_t, u_{t-1})} \left\{ u_t \left( \alpha Z_t + (1 - \alpha)Z_t^+ \right) - (1 - u_{t-1})X_t \right\} + E_t^Q V_{t+1}(G, P, u_t) \]

\[ = \max \left\{ u_{t-1} \left( \alpha Z_t + (1 - \alpha)Z_t^+ \right) + E_t^Q V_{t+1}(G, P, u_{t-1}), \right\} \]

\[ (1 - u_{t-1}) \left( \left( \alpha Z_t + (1 - \alpha)Z_t^+ \right) - X_t \right) + E_t^Q V_{t+1}(G, P, 1 - u_{t-1}) \]
Standard Heuristics and Their Relationship to Optimal Value

- Spread and Heat Rate Options
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- Only considers a specific subset of dispatch decisions (block starts)
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  - Essentially expresses the (incremental) optionality of changes of state
    - Implementation is usually fairly ad-hoc
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  - Remain on at min level, and ramp up to max level if economical
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  - Much richer set of decisions, but still a lower bound, need to determine how suboptimal it is
    - Perfect foresight or lookback value is a possible criteria, but generally provides too high an overestimate to be useful

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Optimal Value Classifications and Martingale Duality

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V_t(G_t, P_t, u_{t-1}) \leq \inf_{M} E_t^Q \max_{u_t, \ldots, u_T} \left[ \sum_{s=t}^{T-1} (H(G_s, P_s; u_{s-1}, u_s) - M_{s+1}^{u_s} + M_s^{u_s}) + H(G_T, P_T; u_{T-1}, u_T) \right]
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Optimal Approximations

Optimal Value Classifications and Martingale Duality

- Notation:
  \[ H(G_t, P_t; u_{t-1}, u_t) = u_t \left( (\alpha Z_t + (1 - \alpha)Z_t^+) - (1 - u_{t-1})X_t \right) \]
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  - Can be used effectively in conjunction with regression-based Monte Carlo or quadrature to improve lower bound valuations
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  - Must also permit diagnostics/reconciliations for checking projections of value-driving, non-traded entities
Sufficient Statistics of the Pricing Problem

- Minimal Martingales
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    - The change of measure (from physical, data–generating measure $P$ to pricing measure $Q$) is in general non–unique
Minimal Martingales

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- Mahoney and Wolyniec 2012c consider the relationship of MMM for general affine processes to heuristic representations such as Black–Scholes
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- Long-term Correlation and Cointegration
  - There is in general a complex interaction of both stationary and non-stationary effects that impact the dynamics of price formation in both the power and gas markets
    - Effects operating at different time scales
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  - Effects operating at different time scales
- These effects have a critical impact on the value that can be extracted from different trading strategies around tolling deals
  - Grzywacz and Wolyniec 2011, Mahoney and Wolyniec 2012a
Sufficient Statistics of the Pricing Problem

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  - The presence of cointegrating effects makes it imperative to distinguish between dynamic and static hedging strategies, and the value that accrues from each kind of strategy
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- Makes commodity (in particular energy) markets very different from financial markets
Sufficient Statistics of the Pricing Problem

- Typical model for gas and heat rate:

\[
\frac{dG}{G} = \mu_g \, dt + \sigma_g \, dw_g
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- Power \((P = HR \cdot G)\) inherits non-stationary effects from fuel used in generation and stationary effects from the fundamental (weather-driven) demand and generation supply stack
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    - Long-term deals may require a blending of dynamic and static value drivers
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- In some cases only a static hedge can be put on, in others some form of dynamic hedging may be possible (*e.g.* Balmo)
- The value that can be collected will typically vary greatly
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    - A very few have quoted heat rate option prices
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  - A very few have quoted heat rate option prices

The relevant point here is that not only does the blending of leg volatility effects impact correlation, but correlation may not be the appropriate value driver at all!

- Eydeland and Wolyniec 2012
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Summary

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Summary

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  - Determines the availability and character of instruments needed to hedge and extract value
- Models need to reflect this structure
  - Understand the nature of the approximations used
  - Be able to connect models to projection of value drivers
References

References

- Mahoney D, and Wolyniec K, 2012b
- Mahoney D, and Wolyniec K, 2012c
- Mahoney D, and Wolyniec K, 2012d